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Elastic wave propagation through a distribution of dislocations

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Abstract

We study the coherent propagation of an elastic wave in a two-dimensional continuous elastic medium filled with dislocation arrays randomly distributed and oriented in space. This configuration reasonably mimics grain boundaries in polycrystals. Interest is in evaluating the plastic contributions to the multiple scattering of waves in polycrystals that may superpose to other known scattering processes, like scattering due to inhomogeneities of elastic properties among grains. Calculations are performed in a multiple scattering formalism, based on the derivation of the so-called mass operator, in the approximation of weak scattering. We find that sound attenuation increases when the frequency decreases, a trend opposite to the usual behavior, suggesting that dislocations could sensibly modify the acoustic properties of materials at low frequency.

Keywords: Dislocations; Grain boundary; Polycrystal; Multiple scattering

1. Introduction

In recent papers [1,2], the acoustic wave propagation through dislocations has been investigated. Ultra-sound techniques offer the possibility of non-destructive testing of materials. Very recent visualizations showing the strong scattering of an acoustic wave by dislocations [3] give further incentives for studying related problems.

One may think that the scattering is due to the local modification of the elastic properties in the vicinity of the defect, that is in the dislocation core. Actually, such a static mechanism is not relevant since the dislocation core size is typically of few nanometers, much smaller than the micro-to-millimeter wavelengths of typical ultrasonic waves. The scattering mechanism is rather a dynamical process that involves two steps [4,5]. The first step is that dislocations are not immobile but can move under the influence of an external stress, here produced by the incident wave. The second step is that an oscillating dislocation produces an oscillating variation of the velocity field that corresponds to an outgoing scattered wave.

This basic mechanism being described by both an equation of motion for the dislocation [6] and an integral representation of the scattered wave for a moving dislocation [7], the scattering properties can be theoretically studied. The scattering of elastic waves by a single dislocation has been studied in [1]. When many dislocations are present, as in real materials, an effective medium approach can be derived from a standard multiple scattering formalism [2]. The goal is to determine the modification of the properties of an incident wave propagating through a random distribution of dislocations, or, in other words, to determine the properties of the so-called coherent (or effective) wave.

This paper extends the work of ref. [2] to the case of the multiple scattering process produced by a random distribution of lines holding a linear density of edge dislocations. Lines of dislocations provide a reasonable picture of low angle grain boundaries, and are expected to give a qualitative description of other boundaries, as far as acoustic properties

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are concerned. Waves propagating in a polycrystal can also be scattered by variations of the bulk elastic properties from one grain to the other. This mechanism, which has been studied for randomly oriented grains and generalized to more complicated textures [8], will be ignored here. In the present paper, we rather take explicitly into account the elastic properties of grain boundaries. In order to analyze separately the contribution of interfaces to the scattering process, we will consider the bulk of the grains as an isotropic continuous medium.

2. Basic equations

The interaction between a single dislocation and an elastic wave, in a 2D geometry, has been described in [2] through Eq. (1):

$$[\nabla^2 + k_\beta^2 + (\gamma^2 - 1)\nabla\nabla.]\mathbf{v} = -V^D\mathbf{v}. \tag{1}$$

Eq. (1) has a classical form: the left hand side term corresponds to the usual wave equation for the time derivative $\mathbf{v}$ of the elastic displacement, whose solutions are two in-plane waves, a transverse wave k_β with velocity $\beta = \sqrt{\mu/\rho}$ and a longitudinal wave k_α with velocity $\alpha = \sqrt{(\lambda + 2\mu)/\rho}$, and $\gamma = \alpha/\beta$. (λ, μ) are the Lamé's constants and ρ the density of the elastic medium. The right hand side term in Eq. (1) describes the interaction between these two waves and the dislocation (i.e. the scatterer) through the potential V^D. The expression of V^D for a gliding edge dislocation moving along its in-plane Burgers vector $\mathbf{b}$ has been derived in [2] in a local basis $(O, \mathbf{t}, \mathbf{n})$, where $\mathbf{b} = b\mathbf{t}$. Defining the matrix $J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, one can express V^D in a more tractable form in a fixed frame $(O, \mathbf{e}_1, \mathbf{e}_2)$ as Eq. (2), where $\mathbf{X}$ is the dislocation position, m the classical effective mass of edge dislocations, $F_{\theta_0} = R_{\theta_0} J R_{-\theta_0}$ and with $\theta_0 = (\widehat{\mathbf{e}_1, \mathbf{b}})$ and R_a the rotation matrix of angle a.

$$V^D(\mathbf{x}) = \frac{\mu b^2}{m\omega^2} F_{\theta_0} \nabla\delta(\mathbf{x} - \mathbf{X})\, {}^t\nabla_{|\mathbf{X}}\, F_{\theta_0}, \tag{2}$$

where ^{t} denotes the transpose. The potential V corresponding to a line distribution of dislocations on the line L with a density ρ_b is obtained by summing over the dislocations:

$$V(\mathbf{x}) = \frac{\mu b^2}{m\omega^2}\rho_b \int_L \mathrm{d}X\, F_{\theta_0} \nabla\delta(\mathbf{x} - \mathbf{Y})^t \nabla_{|\mathbf{Y}}\, F_{\theta_0}, \tag{3}$$

where $\mathbf{Y} = \mathbf{X}_c + \mathbf{X}$, with $\mathbf{X}_c$ the origin point on L and $\mathbf{X}$ oriented along L (Fig. 1).

3. The modified Green function

The multiple scattering formalism is based on the calculation of the modified Green function $\langle G\rangle(\mathbf{k})$. The modified Green function describes the elastic medium filled with scatterers randomly distributed (here, the segments L representing grain boundaries) in terms of an effective medium. The averaging process over disorder realizations involves a priori averages over the lengths L of the segments, over the dislocations densities $\rho_b = 1/d$ held by each line, over the Burgers vectors b of the $N = \rho_b L$ dislocations held by L, and over the positions and orientations of the segments $(\mathbf{X}_c, \theta_0)$ (Fig. 1). The modified Green function is given by the Dyson equation:

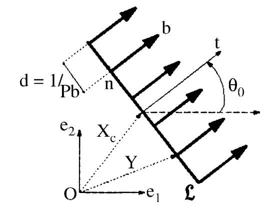

Fig. 1. Grain boundary pictured by a line L, of length L with density ρ_b of gliding edge dislocations of Burgers vector $\mathbf{b}$.

$$\langle G\rangle(\mathbf{k}) = [G^{0^{-1}}(\mathbf{k}) - \Sigma(\mathbf{k})]^{-1}, \tag{4}$$

where G^0 is the disorder-free Green's function and $\Sigma(\mathbf{k})$ the so-called mass-operator. In the weak scattering (or weak disorder) limit, measured by a small parameter ϵ, $\Sigma(\mathbf{k})$ can be expanded in power of ϵ, $\Sigma(\mathbf{k}) = \Sigma_1(\mathbf{k}) + \Sigma_2(\mathbf{k}) + \ldots$. In the present case, we need to compute at least the first two terms, because the imaginary part of the leading term $\Sigma_1(\mathbf{k})$ vanishes. These terms are given by

$$\Sigma_1(\mathbf{k}) = n \int \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{C}\, \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}} V(\mathbf{x})\, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}},$$
$$\Sigma_2(\mathbf{k}) = n \int \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{x}'\, \mathrm{d}\mathbf{C}\, \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}} V(\mathbf{x}) G^0(\mathbf{x} - \mathbf{x}') V(\mathbf{x}')\, \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}'}, \tag{5}$$

where n denotes the number density of scatterers and where the integral over $\mathbf{C}$ corresponds to averages over all relevant parameters. Here $\mathrm{d}\mathbf{C} = p(b)\,\mathrm{d}b\; p(L)\,\mathrm{d}L\; p(\rho_b)\,\mathrm{d}\rho_b \mathrm{d}\mathbf{X}_c/V\,\mathrm{d}\theta_0/(2\pi)$, where $p(X)$ denotes the probability distribution function of the quantity X. In Eq. (5), we have assumed that the scatterers are not spatially correlated. The (complex) poles of $\langle G\rangle(\mathbf{k})$ give the wavenumbers K_α and K_β of the coherent waves susceptible to propagate in the effective medium. Their real part is related to the index of refraction whereas their imaginary part to the attenuation length. In order to simplify the calculations, we adopt the following method. The modified Green's function $\langle G\rangle(\mathbf{k})$ is expressed as a function of $\langle\tilde{G}\rangle(k)$ [Eq. (6)], the modified Green function in the local reference frame $(O, R_\xi\,\mathbf{e}_1, R_\xi\,\mathbf{e}_2)$ where $\xi = (\widehat{\mathbf{e}_1, \mathbf{k}})$. In this frame, we expect $\langle\tilde{G}\rangle(k)$ to be diagonal and to depend only on the modulus k of $\mathbf{k}$.

$$\langle G\rangle(\mathbf{k}) = R_\xi\, \langle\tilde{G}\rangle(k)\, R_{-\xi}. \tag{6}$$

Using the Dyson equation (4), our task is to compute $\langle\tilde{G}\rangle(k)$ as

$$\langle\tilde{G}\rangle(k) = [\tilde{G}^{0^{-1}}(k) - \tilde{\Sigma}(k)]^{-1}, \tag{7}$$

with $\tilde{\Sigma} = \tilde{\Sigma}_1 + \tilde{\Sigma}_2$ at second order in disorder strength, and with

$$\tilde{G}^0(k) = \begin{pmatrix} \dfrac{1}{\gamma^2(k^2 - k_\alpha^2)} & 0 \\ 0 & \dfrac{1}{(k^2 - k_\beta^2)} \end{pmatrix}. \tag{8}$$

After some calculations [9], we obtain

$$\tilde{\Sigma}_1(k) = -\frac{1}{2}\frac{\mu\, n}{\omega^2}\left\langle \frac{N\, b^2}{m} \right\rangle k^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\tilde{\Sigma}_2(k) = \frac{i}{16}\left(\frac{\mu\, n}{\omega^2}\right)^2 \left\langle \frac{N^2 b^4}{m^2} \right\rangle \frac{1+\gamma^4}{\gamma^4}\frac{k_\beta^2}{n} k^2 \times \begin{pmatrix} I_1(kL) & 0 \\ 0 & I_2(kL) \end{pmatrix} \tag{9}$$

with $I_a(kL) = \int \frac{p(L)\, L^2\, \mathrm{d}L\, \mathrm{d}\theta_0\, \mathrm{d}\zeta}{\pi^2 \langle L^2 \rangle (1+\gamma^4)} f_a(\theta_0) F(L, \theta_0, \zeta)$ for $a = 1, 2$, $f_1 = \sin^2 2\theta_0$, $f_2 = \cos^2 2\theta_0$, $F = \cos^2 2\zeta f(k_\alpha L, \theta_0, \zeta) + \gamma^4 \sin^2 2\zeta f(k_\beta L, \theta_0, \zeta)$, and $f(qL, \theta_0, \zeta) = \sin c^2[(k \sin\theta_0 - q \sin\zeta)L/2]$.

4. Characteristics of the coherent waves

The wavenumbers K_α and K_β of the coherent longitudinal and transverse waves, respectively, are given by the poles of $\langle G \rangle^{-1}(\mathbf{k})$, or equivalently by the poles of $\langle \tilde{G} \rangle^{-1}(k)$, defined in (6) and (7) and calculated above.

4.1. Index of refraction

We define the index of refraction as $n_a = v_a / V_a$ for $a = \alpha, \beta$, where $V_a = \omega / \Re(K_a)$ and $v = \omega / k_a$ denote the phase velocities, respectively, in the presence and in the absence of scatterers. From above, we obtain for $c = \alpha, \beta$ and with $a_\alpha = 1, a_\beta = \gamma^2$,

$$n_c = 1 + \frac{a_c}{4\gamma^2}\frac{\mu\, n}{\omega^2}\left\langle \frac{Nb^2}{m} \right\rangle \tag{10}$$

As observed for a distribution of isolated dislocations [2]

- The effective phase velocity is larger than its value in the absence of scatterers. Reversely, the group velocity is smaller.
- The index of refraction increases with increasing the wavelength.

The latter behavior is unusual: for static inhomogeneities, scattering is expected to vanish at long wavelengths, and the index to decrease toward unity. As first observed by Nabarro [5] and confirmed in our calculations, this result is due to the particular dynamical interaction between an elastic wave and a dislocation: the scattering process occurs because the motion of the dislocation is driven by the incident wave, motion whose amplitude increases with increasing the wavelength [6].

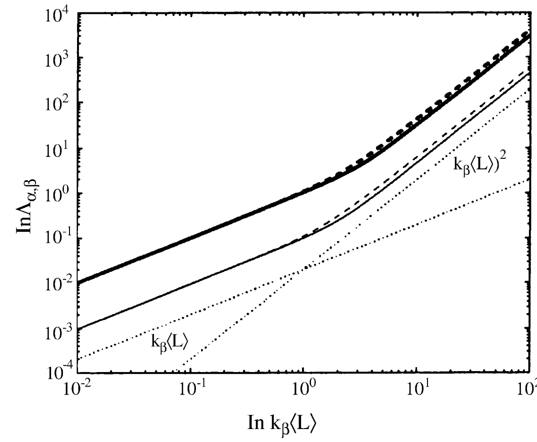

Fig. 2. Λ_α (bold line) and Λ_β (thin line) vs. $k_\beta\langle L\rangle$ ($\gamma = 1.4$), for a distribution of length $p(L) \propto \delta(L - \langle L \rangle)$ (solid line), and $p(L) = 1/(2\langle L \rangle)$ (dashed line). The straight dashed lines are guides to the eye.

4.2. Attenuation lengths

The attenuation length $\Lambda_a \equiv 1/\Im(K_a)$ corresponds to the loss of energy due to scattering away from the forward direction. We obtain, for $c = \alpha, \beta$

$$\Lambda_c = \frac{32\gamma^4}{1+\gamma^4}\frac{c^4}{n\mu^2}\left\langle \frac{m^2}{N^2 b^4} \right\rangle \frac{k_c}{I_c(k_c L)} \tag{11}$$

with $I_\alpha = I_1$, $I_\beta = I_2$ (see Fig. 2).

Increasing the wavelength makes the attenuation length $\Lambda_{\alpha,\beta}$ decrease, again an unusual behavior for waves propagating in random media. Note the presence of a linear and a quadratic regime, with a cross-over between both behaviors occurring at wavelengths of the order of the average grain boundary length.

Finally, in the limit $k_c L \ll 1$, the expressions of the refraction index and the attenuation length equal those found for a distribution of single dislocation, with total mass Nm and Burgers vector Nb (see [2]).

5. Concluding remarks

The dispersion relation of an elastic medium filled with dislocation arrays randomly distributed and oriented in space has been derived. This analysis was aimed to evaluate the plastic contribution to the scattering of elastic waves propagating in a polycrystal. It has been found that the strength of this scattering phenomenon increases when decreasing frequency, a result that has to be contrasted with the usual behavior observed when the primary source of scattering is due to bulk anisotropy [8]. Both effects may superpose in polycrystals, and the plastic one could be important at low frequency.

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