



Comment

What future for Lévy walks in animal movement research?  
Comment on “Liberating Lévy walk research from the shackles  
of optimal foraging”, by A.M. Reynolds

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Due to the apparent erratic nature of animal trajectories, the standard random walk, including variants like the correlated random walk (CRW), represents a natural framework for modeling animal movement [1]. The complexity of movement decisions is reduced to a stochastic noise for the sake of simplicity. This approach can be further simplified thanks to the Markov hypothesis, which is very popular. Namely, if one also assumes that successive displacements are independent, or if memory decays sufficiently fast over time like in the CRW, the theory of random walks becomes relatively friendly and easy to apply. Not less relevant to animal mobility are non-Markovian processes, though, in particular self-repulsing and self-attracting random walks, which commonly model ants and bacteria movements [2]. These stochastic processes, arguably more realistic in many situations and with rich emerging properties, have not found their place (yet?) in the dominant animal movement paradigm. More because of their formidable mathematical complexity than their lack of relevance, probably.

In this context, the introduction of Lévy flights (LFs) in foraging theory [3,4], in spite of its importance, was not a revolution. Lévy flights are Markovian, memoryless processes and completely adhere to the classical view of erratic trajectories advocated by Turchin [1]. Still, they offered a new tool for modeling with very few parameters movement at multiple scales, an heterogeneous occupation of space, or faster-than-Brownian diffusion (these features bringing possible advantages as a foraging strategy). The conceptual oddities associated to LFs, such as the persisting discontinuities and the presence of infinite quantities resulting from unbounded power-laws, have raised many concerns regarding their relevance to the real world. One should not forget, however, that the same problems are encountered in fractals, a widely accepted description of numerous natural patterns [5]: we perfectly know that no such thing like

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a genuine, infinitely self-similar fractal exists in nature, which obviously does not mean that this type of geometry is unrelated to real phenomena. Similar considerations can apply to Lévy flights.

What is really a Lévy flight? The question is not as incongruous as it may sound, since LFs appear to mean different things in different places. Following Paul Lévy himself, what is normally called a Lévy distribution is a fairly peculiar function: a distribution which remains stable by summing and rescaling in a particular way independent random variables drawn from it [6]. (Typically, the summands are the displacement steps and the sum is the position.) It is an emerging property, much like Gaussians are stable and emerge by adding random variables of finite variance. Lévy walks are more complicated than LF, but not extremely different. In recent years, due to its increasing popularity, one is forced to notice an abuse of the term “Lévy distribution”: very often it is a mere label for a distribution of some quantity which behaves as a power-law at large arguments. Yet, if the quantity of interest is not related to a problem of sums of random variables, making a connection with the (quite complicated) functions found by Lévy is at best a vague analogy.

This sloppy use of terms is not a minor issue: it has obscured the merits LF modeling, and generated confusions which are not unrelated with several recent controversies [7]. One can also notice that many studies on biological LFs have been confined in recent years to statistical discussions [8–10]. Comparatively, biological and physical advances have been scarce [11]. For instance, is it that important to determine that a dataset is best fitted by a composite CRW than a LF? Both processes produce very similar patterns in practice, as rightly pointed out by Reynolds [12]. Instead, isn't it more important to unveil a new biological function conferred by the heterogeneity of the movements in a system of study?

One must credit Reynolds for stressing that a variety mechanisms can generate power-law patterns (refraining here from using the word Lévy) [12]. But if very different models predict more or less the same power-laws, the former are not very useful unless one has alternate measures at hand to discriminate between interpretations. A lot of future research efforts are still needed in that direction. Beyond model comparison, the limits of a given framework can be addressed through model validation: can a fitted model make additional predictions, also observed in the field? Not surprisingly, CRW and LF have many limitations, the price to pay for simplicity, since random walks hardly capture the behavioral complexity of many animals and the ways they may interact with their environments.

Whereas it is far from obvious that “Lévy walk research” constitutes a well-defined field, animal movement research is blossoming and Lévy walks may have a role to play in its future developments. Data on animal movement are accumulating at great speed, yet, our understanding does not grow as fast. Very recently, in a different context, a renewed attention has been paid to self-organized criticality (SOC), a trendy concept that emerged in the 80s and that was presented by its supporters as able to explain the ubiquity of power-law distributions (in evolution, earthquakes, financial time series, etc...). SOC was the source of many controversies. P. W. Anderson, the Nobel prize laureate in condensed matter physics, gave the following point of view [13,14]: “SOC seems to me to be not the right and unique solution to these and other similar problems, but to have paradigmatic value, as the kind of generalizations which will characterize the next stage of physics.” A comment applicable to the Lévy flight foraging hypothesis, perhaps.

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