

# Lévy-like behaviour in deterministic models of intelligent agents exploring heterogeneous environments

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## Abstract

Many studies on animal and human movement patterns report the existence of scaling laws and power-law distributions. Whereas a number of random walk models have been proposed to explain observations, in many situations individuals actually rely on mental maps to explore strongly heterogeneous environments. In this work, we study a model of a deterministic walker, visiting sites randomly distributed on the plane and with varying weight or attractiveness. At each step, the walker minimizes a function that depends on the distance to the next unvisited target (cost) and on the weight of that target (gain). If the target weight distribution is a power law,  $p(k) \sim k^{-\beta}$ , in some range of the exponent  $\beta$ , the foraging medium induces movements that are similar to Lévy flights and are characterized by non-trivial exponents. We explore variations of the choice rule in order to test the robustness of the model and argue that the addition of noise has a limited impact on the dynamics in strongly disordered media.

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## 1. Introduction

Deterministic walkers, as opposed to random ones, follow non-stochastic rules of motion. They can be used as an approximation to describe mobile agents that (i) have some information on the medium they explore (composed, for instance, of food patches, facilities or cities, depending on the context) and (ii) use this information to optimize their gain (such as food

intake or the possibility of performing a particular activity). If the medium is disordered and heterogeneous, deterministic rules can produce fairly erratic and complex trajectories, susceptible to being studied statistically as random walks.

Unlike Brownian particles, humans and other animals keep a memory of their past activity. For example, a class of deterministic walks assumes that each decision depends on the previous steps the walker has performed. This kind of process can be applied to a wide range of hard-to-solve problems such as the travelling salesman or the travelling tourist [1, 2]. Yet, despite their potential importance in a variety of problems in physics, mathematics, computer science, economics and biology, the properties of deterministic walks remain largely unexplored. Deterministic diffusion has been investigated in simple random media [3, 4]. Similar processes can be used to implement classification schemes [5, 6] and for pattern recognition [7]. In biology, though not widely used, deterministic walks offer an appealing framework to model the movement of foraging animals. Indeed, many organisms rely on mental maps to navigate their environment in a non-random way [8]. Primates and other animals dispose of sophisticated cognitive skills [9], such as the ability to remember autobiographical events that happened at some particular places and times (episodic memory).

The presence of scaling laws and power-law distributions in animal movement patterns is an ecological problem that has attracted considerable attention in the last decade [10]. Empirical evidence of Lévy walks has been reported, among others, for micro-organisms [11], bumblebees [12], spider monkeys [13] and marine predators [14]. Similar patterns have also been observed in humans, as shown by studies on hunter-gatherers [15], bank notes [16] or fishermen boats [17]. Human individuals also display multiple scale displacements well described by truncated power laws [18]. Most of the theoretical interpretations proposed to explain observations rely on the random walk hypothesis or some variant thereof: for example, the diffusion of bank notes is actually well described by a continuous time random walk model [16].

A common hypothesis is that the ubiquitous presence of widely fluctuating characteristic scales in animal movement could optimize the success of search processes of randomly distributed resources [12]. According to this perspective, animals would execute a particular optimal foraging strategy, usually a Markovian stochastic process (see [12, 19, 20] for examples), to find prey that are uniformly distributed and only detectable at short distance. These strategies are susceptible of being learned along an individual life experience [14], or transmitted through generations as an evolutionary trait [21].

An alternative hypothesis, namely, that scale-free (or more generally, multiple scale) movement patterns may be an outcome of the direct interaction of the forager with a complex environment, has been much less explored quantitatively. In this approach, very simple movement rules, sometimes involving memory, may produce complex searching patterns. For instance, a simple response to passive scent concentrations in turbulent structures can account for scale-free trajectories of flying insects [22]. Lévy patterns similar to those observed in jackals can also arise from avoidance of conspecific odour trails, as modelled by systems of self-avoiding walks [23]. Lévy-like distributions of the distances between detected preys can also emerge in the trajectories of a predator following, deterministically, chemotaxis gradients produced by immobile and randomly located prey [24].

In this paper, we study a model describing the movement of a walker that forages in a strongly heterogeneous environment using mental maps. It is assumed that the forager has a perfect knowledge of the available resources (for instance, fruiting trees) and that it chooses deterministically at each step the ‘best’ (in terms of an efficiency function) unvisited food patch in the whole system. Therefore, decisions are not taken on the basis of the knowledge of the immediate surroundings only. We originally proposed this model in the context of a study

on the ranging patterns of spider monkeys in tropical forests [25, 26], but we expect it to have a much wider application range. In these works, we showed that scale-free distributions of step lengths can emerge if the resources are sufficiently diverse in size. Here we give a detailed analysis of the model, we show that the behaviour of the walker is robust to the addition of noise and we study how the results are affected by changes in the deterministic decision rules.

## 2. The model

Consider a two-dimensional square domain of unit area, randomly and uniformly filled with  $N$  point-like targets of fixed positions. The targets model, for instance, fruiting trees in an ecological context, or cities in a social one. The targets may also represent network communities, nodes for information retrieval or markets for salesmen. Each target  $i$  is characterized by, say, a food content or a size,  $k_i$ , which are independent random integers drawn from a distribution  $p(k)$ . (Integers are considered here for computational convenience, without restricting generality.) The distributions of tree sizes in tropical and temperate forests are very broad and in many cases well described by inverse power laws with exponents ranging from 1.5 to 4 [27, 28]. Across species, the size of a tree is roughly proportional to its fruit mass [29]. We will therefore consider target size distributions of the power-law form:

$$p(k) = Ck^{-\beta}, \quad \text{with } k = 1, 2, \dots, k_{\max}; \quad \beta > 1, \quad (1)$$

$C$  being the normalization constant and  $k_{\max}$  is a cutoff size that can be taken infinite (see section 6) but will be fixed to  $10^3$  in the simulations below, except when otherwise indicated. In the social context, (1) with  $\beta = 2$  is equivalent to assuming that city sizes are distributed according to Zipf's law [30].

In this domain, a forager (or traveller) is initially located on a target. We assume that the forager has a perfect knowledge of the size and position of every target in the system. In a human context, when the travellers are, say, salesmen or tourists this assumption is natural. On the other hand, in a biological context, this assumption is based on observations that many animals (primates [8, 9], birds [31]) use local and global cues during searching. Field studies show that Clark's nutcracker (among other species of small birds) can store 22 000 to 33 000 nuts in up to 2700 locations over an area larger than 100 square miles and remember where almost 70% of them were placed [32]. One could also relax the perfect knowledge hypothesis by assuming that the forager knows a random subset of  $N_s$  targets, or the  $N_s$  largest targets ( $1 \ll N_s < N$ ). These known targets would still be described by the distribution (1) with exponent  $-\beta$ .

At each step ( $n \rightarrow n + 1$ ) the following rules are iterated:

- (i) The walker located at target  $i$  will next visit the target  $j$  such that the step efficiency  $E_{ij}$  is maximum among all allowed  $j \neq i$  in the system.
- (ii) Previously visited targets are not revisited (which corresponds to destructive foraging in the ecological context).

Presently, we will consider two forms for the efficiency function  $E_{ij}$ :

$$E_{ij} = k_j/l_{ij} \quad (\text{'quotient' rule}), \quad (2)$$

$$E_{ij} = k_j - l_{ij}/l_0 \quad (\text{'difference' rule}), \quad (3)$$

where  $k_j$  is the size of target  $j$ ,  $l_{ij}$  is the distance separating targets  $i$  and  $j$ , and  $l_0 = 1/\sqrt{N}$  is the typical distance between nearest neighbour targets. Hence, in both rules (2) and (3), the forager tends to visit the largest possible target accessible in a minimum travelled distance

from its current position, ignoring targets that are either too small or too far away. Rule (3), which has not been studied before in [25, 26], represents the net energy balance of a sojourn, assuming that the energy expenditure is proportional to the distance travelled. Note that in this case we do not impose the condition  $E_{ij} > 0$ : some steps may be such that energy expenditure exceeds energy gain (see section 5).

### 3. Step lengths statistics

In [26], a mean field, single-step solution of the model for the quotient rule (2) was obtained in any spatial dimension  $d$ . There, it was shown that the heterogeneity of the medium can induce power-law distributions  $P(l)$  of sojourn lengths (distances between the origin and the best target in that case): in  $d = 2$ , if  $\beta \geq \beta_c = 3$  in the target size distribution (1), then

$$P(l) \propto l^{-\alpha_{MF}}, \quad \text{with} \quad \alpha_{MF} = \beta - 2. \quad (4)$$

Therefore displacements can be considered as Lévy-like ( $1 < \alpha_{MF} < 3$ ) if  $3 < \beta < 5$ . If  $\beta < 3$  and  $k_{\max} = \infty$ , the density of very big targets in the medium is so high that  $P(l)$  is no longer scale-free and involves a characteristic size diverging with  $N$  [26].

Although the mean-field description captures the gross behaviour of the model, it fails to explain the numerical results quantitatively (see below). By imposing the condition that the walker cannot return to a previously visited site, memory effects produce non-trivial long-range correlations between steps as the process develops. In [25],  $2d$  simulations with the quotient rule (2) indeed exhibit scaling-laws, but different exponents are found:

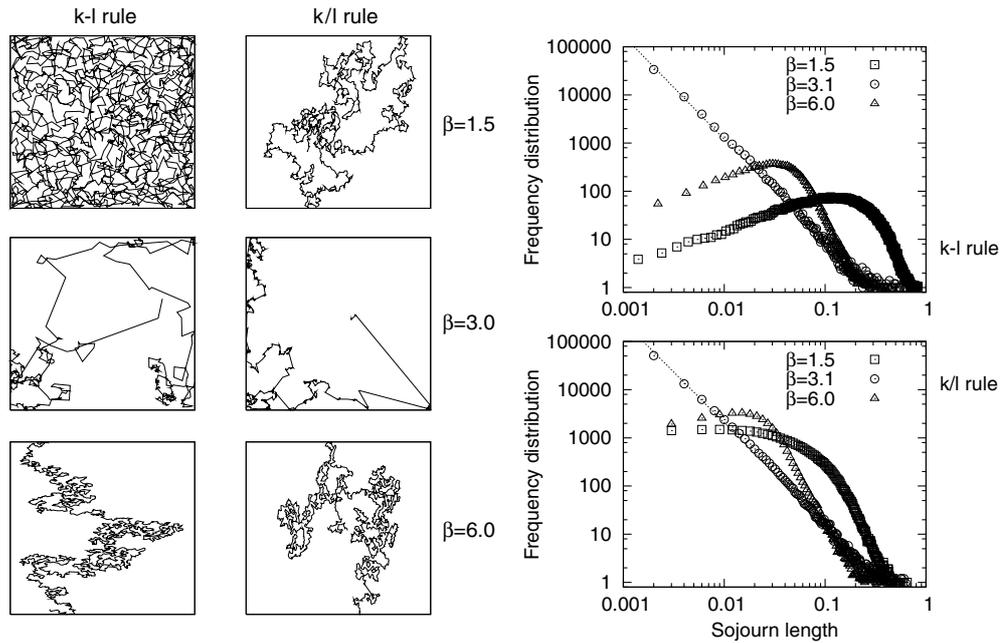
$$P(l) \propto l^{-\alpha}, \quad \text{with} \quad \alpha \simeq \beta - 1 (\neq \alpha_{MF}), \quad \text{for} \quad 3 \leq \beta \leq 4. \quad (5)$$

We discuss in more detail the numerical results below.

Several trajectories are represented in figure 1 (left), with the corresponding step length distribution in figure 1 (right). With the ‘difference’ rule (3), three qualitative regimes can be identified as  $\beta$  is varied. In the first one (superabundant regime), corresponding to  $\beta < 3$ , the trajectories are dominated by large sojourn lengths. If  $\beta$  is small enough, there are many attractive (large) targets, so that the forager often travels large distances to reach them. Very large targets are relatively distant from each other, but the large values of their size  $k$  make these long sojourns ‘affordable’. The situation changes dramatically in the interval  $3.1 < \beta < 4.0$  (diverse regime), where very big targets exist but are too scarce to produce frequent long travels. As a result, there are a few large sojourn lengths alternated with many shorter ones at all scales. In this regime, the distribution of step lengths is a power law with an exponent  $\alpha$  approximately given by relation (5). Trajectories in this regime closely resemble Lévy flights. Finally, when  $\beta > 4$  (homogeneous regime), target size fluctuations are too small and the forager performs mostly short steps to nearby targets, similar to a (self avoiding) random walker.

The three regimes discussed above, in particular the scaling regime with  $\alpha \simeq \beta - 1$ , see (5), are also observed when the walker follows the ‘quotient’ rule (2). In the superabundant regime, nevertheless, the steps are on average shorter than for the difference rule (see figure 1, right).

Further insight into the statistical properties of the walks can be gained by studying step length fluctuations. Figure 2 displays the fluctuation ratio, defined as  $\langle l^2 \rangle / \langle l \rangle^2$ , versus the resource exponent  $\beta$ . Fluctuations are maximum at  $\beta_c \approx 3$  (independently of  $k_{\max}$ ) for both decision rules, confirming the results obtained in [25] for rule (2) only. Therefore, a medium with  $\beta = \beta_c$  produces, in some sense, the most scale-free walks that are characterized by



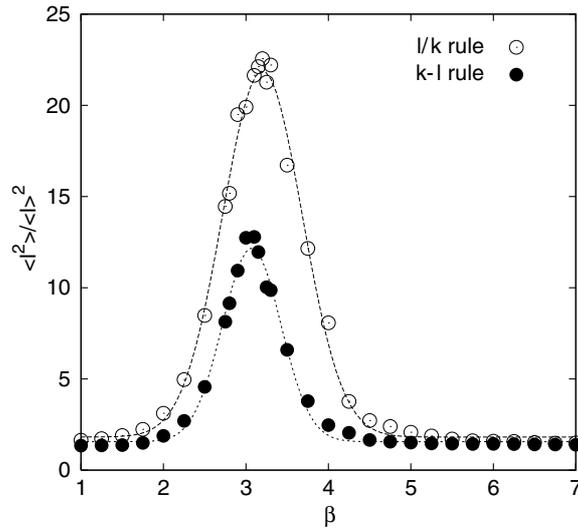
**Figure 1.** Left panels: examples of deterministic trajectories for three values of the target size distribution exponent:  $\beta = 1.5$  (superabundant medium), 3 (diverse medium) and 6 (homogeneous medium). When  $\beta = 3$ , the sojourn length fluctuates widely in both rules, similarly to a random Lévy flight. ( $N = 10^6$ , number of steps per walk =  $10^3$ .) Right panels: step length distributions. Near-perfect power-law distributions,  $P(l) \sim l^{-\alpha}$ , are obtained in the range  $3 \leq \beta \leq 4$ . The value of the scaling exponent at  $\beta = 3.1$  is  $\alpha \simeq 2.1$ , while the value at  $\beta = 4.0$  is  $\alpha \simeq 3.0$ .

$\alpha \simeq 2$ . As shown in figure 3,  $\beta_c$  also corresponds approximately to the onset of emergence of very large steps:  $(l_0/\langle l \rangle)^2$  becomes very small below  $\beta_c$ .

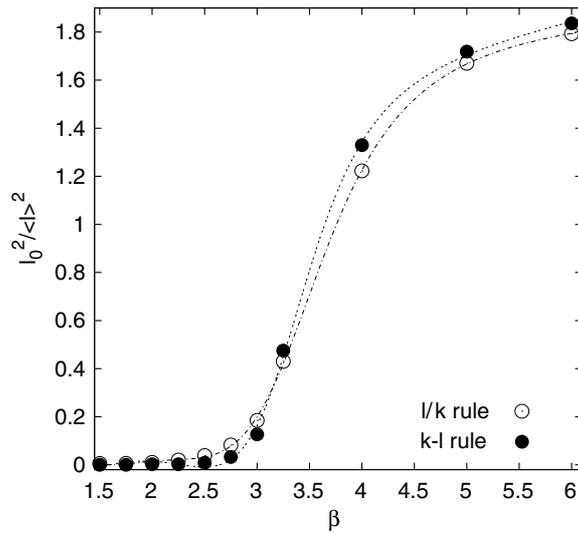
#### 4. Sizes of visited sites and non-stationarity

We now analyse the size of visited targets, which is related to the intake of the forager in the ecological context. Figure 4 shows the temporal evolution of the average size  $\langle k_i \rangle$  of the  $i$ th visited target, for the  $k - l/l_0$  rule and a superabundant medium with  $\beta = 2.25$ . For this value of  $\beta$ , it is clear that this quantity is not stationary. Three qualitative stages can be distinguished. The initial stage is characterized by the visit to large targets all of which are relatively similar in size; in a following stage, the size of visited targets decreases slowly with time; in the final stage (where still 99% of the targets have not been visited) the visited targets have small sizes.

This behaviour is typical of the superabundant regime ( $\beta < \beta_c$ ) and can be understood qualitatively in terms of the length scales considered by the forager. In the initial regime, decisions are taken essentially in terms of the target size, the distance travelled playing a minor role (see section 6 for an analytical argument). In this regime, large targets are so big that the walker chooses them in spite of having to take long steps to reach them, steps that can actually be of the order of the system size. Once the large targets have been depleted, there are no targets left that are worth travelling distances of the order of the system size, but there

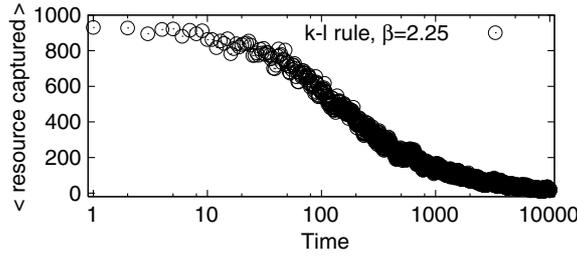


**Figure 2.** Length fluctuation ratio,  $\langle l^2 \rangle / \langle l \rangle^2$ , as a function of the resource exponent  $\beta$ . In both the  $k-l/l_0$  and  $k/l$  rules, this quantity is maximum at  $\beta \approx 3.1$ . In all cases,  $N = 10^6$  and  $k_{\max} = 10^3$ . Data were obtained from averaging over 10 different disordered media. In each realization, a walk starts near the centre of the square domain and visits  $10^5$  targets, a number still  $\ll N$ .

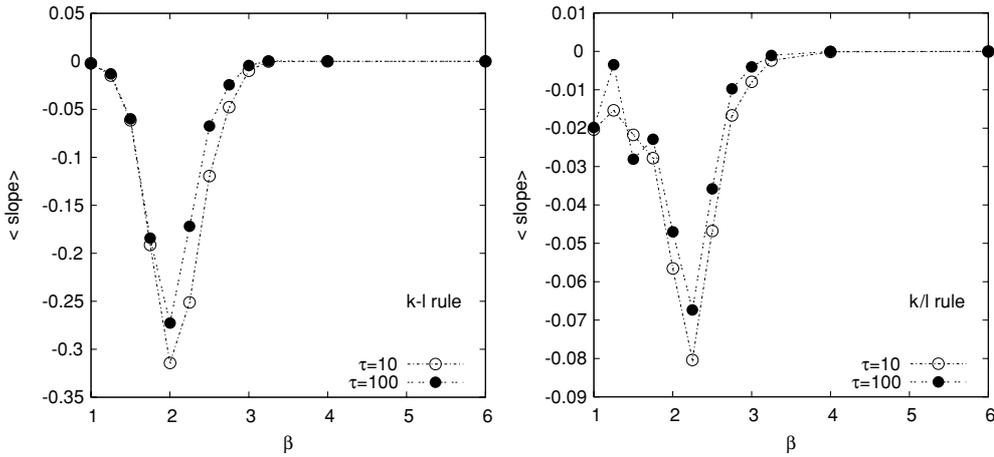


**Figure 3.**  $l_0^2 / \langle l \rangle^2$  as a function of  $\beta$  showing diverging steps below  $\beta_c$ .  $k_{\max} = 10^6$ , other system settings as in figure 2.

are still plenty of targets worth travelling distances which are much longer than the typical inter-target distance  $l_0$ . Finally, in the last regime most of the valuable targets have already been depleted, and the walker chooses among the many small targets that are at distances of the order of  $l_0$ .



**Figure 4.** Temporal evolution of the average size of the  $i$ th visited target, in the  $k-l$  rule and with  $\beta = 2.25$ ,  $N = 10^6$ ,  $k_{\max} = 10^3$ . Averages are taken over ten independent runs.



**Figure 5.** Average time derivative  $\langle \bar{k}_{i+1} - \bar{k}_i \rangle_i$  performed over the time domain  $i = 1, \dots, 10^4$  (for the rules  $k-l/l_0$  and  $k/l$ ) as a function of  $\beta$ . Each point is the average of ten replicas. The quantity  $\bar{k}_i$  is defined as the average value of  $\tau$  consecutive  $k_i$  values. Note the onset of stationarity at  $\beta = 3$ .

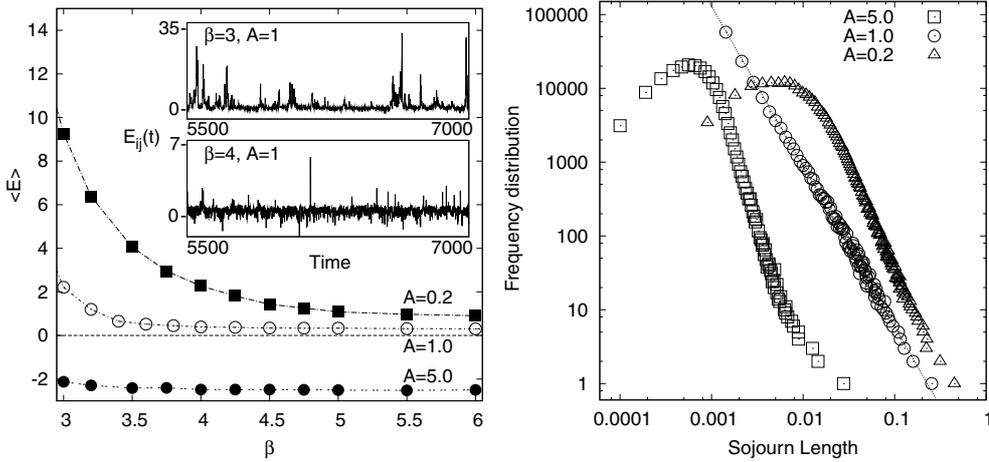
Figure 5 shows the average decay rate of  $\langle k_i \rangle$ . It is interesting to note that the process becomes essentially stationary for  $\beta > \beta_c$ . Therefore, in the scaling regime (5), even though the forager modifies its environment, the medium explored is statistically independent of time (as long as the number visited target is  $\ll N$ ).

### 5. Energy balance and power laws

Some insights into the properties of the model with the ‘difference’ rule (3) can be obtained by modifying the energy cost associated with travels. We introduce a new parameter,  $A$ , which is the energy spent by the forager per unit length  $l_0$ :

$$E_{ij} = k_j - Al_{ij}/l_0. \tag{6}$$

In general, we expect that different values of  $A$  will induce (in a same medium) changes in the target choices, that will translate into changes in the length of the steps and therefore in their distribution.



**Figure 6.** Left panel: average energy balance as a function of  $\beta$  for three values of  $A$  in rule (6). Insets:  $E_{ij}(t)$  as a function of  $t$  for  $\beta = 3$  and 4 ( $A = 1$  in both cases). Right panel: step length distribution at fixed  $\beta = 3$  and for three values of  $A$ .

The case  $A = 1$  is the one analysed so far. If  $A \gg 1$ , we expect the average energy balance  $\langle E_{ij} \rangle$  to be negative (despite the optimal decisions) and the forager starves. When  $A \ll 1$ , on the other hand, we expect the energy gained to be much larger than the energy consumed in travels. The distances between successive targets should then not play a major role in the decisions. Figure 6, left panel, shows  $\langle E_{ij} \rangle$  as a function of  $\beta$ , in the stationary regime (i.e.  $\beta \geq 3$ ), for  $A = 1, 5$  and  $0.2$ . We observe that the case  $A = 1$  is of particular interest and also biologically relevant, in the sense that  $\langle E_{ij} \rangle > 0$  and  $\langle E_{ij} \rangle = O(1)$  for all values of  $\beta \geq 3$ . These properties do not hold in the other two cases. For  $A = 5$ ,  $\langle E_{ij} \rangle < 0$  for all values of  $\beta \geq 3$ : a forager with such energy expenditure would not survive in these media. For  $A = 0.2$ , we find  $\langle E_{ij} \rangle$  is quite larger than 1 at  $\beta = 3$ .

In the insets of the left panel of figure 6, we display typical time series  $E_{ij}(t)$  for  $A = 1$  and two different values of  $\beta$ . We observe in both cases a stationary process. Remarkably, temporal fluctuations in the case  $\beta = 3$  contain a few bursts of size much larger than the average, a feature reminiscent of a critical phenomenon.

The right panel of figure 6 shows the step length distribution  $P(l)$  at  $\beta = 3$ , for the three values of  $A$  considered above. Only the case  $A = 1$  clearly exhibits a power law ( $\propto l^{-2}$ ). We conclude that power laws emerge in this model from the interplay between the energy cost due to travels (that involves a space variable,  $l_{ij}$ ) and the energy gain ( $k_j$ , which is not a space variable), when both terms are on average of the same order.

## 6. The effect of noise in the choice of a target

We analyse in this section a probabilistic extension of the deterministic decision rule, based on the ‘quotient’ efficiency (2). The approach presents some similarities with that of [33, 34] where a stochastic version of a deterministic walk visiting point-like targets was studied.

In principle, different approaches can be followed to introduce noise in the model. In one such approach, studied in [25], the forager still seeks to optimize its movements but has an imperfect knowledge of the medium (e.g., the values of tree positions and food contents

in the mental map differ from the real ones by some random fluctuating amounts); hence, the efficiency of a step is not evaluated exactly and the trajectory can depart from the ideal deterministic one. For this case, it was shown that in the Lévy regime these forager mistakes have little effect on the step-length distributions [25]. Here we focus on a different approach for introducing noise into the system. In the present approach, the forager still has a perfect knowledge of the medium and correctly evaluates step efficiencies, but it may decide not to visit the best target. This mimics the free-will, so to say, of the agent, allowing for non-optimal choices.

For simplicity, let us consider a one-dimensional lattice with  $N$  regularly spaced targets at coordinates  $i = 1, 2, \dots, N$  and a walker initially located at coordinate 0. Similarly to (1), target weights are quenched and distributed according to

$$p(k) = (\beta - 1)k^{-\beta}, \quad \beta > 1, \quad (7)$$

with  $1 \leq k < \infty$  now a continuous variable. We now assume that the walker chooses a target  $i$  with a probability  $p_i$  that depends on the efficiency  $E_i = k_i/i$  of the step. We assume  $p_i \propto f(E_i)$ , with  $f(x)$  an increasing function of  $x$ . Hence, high-efficiency targets have higher probabilities of being chosen, but the process is not deterministic. For further simplicity, we study the *single* step version of this problem. A somewhat similar diffusion problem has been studied in the weak disorder limit in [35]. Similar ideas were also applied to the study of a glass transition in a kinetic trap model with hopping rates depending on inter-trap distances [33, 34].

Let us denote  $r_i \equiv f(E_i)$ , hence,  $p_i = r_i / \sum_j r_j$ . In this section we calculate the overlap  $Y_2$ , defined as the average probability that two independent walkers located at the origin in the same medium choose the *same* target:

$$Y_2 = \langle y_2 \rangle, \quad \text{with} \quad y_2 = \sum_{i=1}^N p_i^2, \quad (8)$$

where  $\langle \cdot \rangle$  denotes the average over disorder configurations  $\{k_i\}$ . A purely deterministic process corresponds to  $Y_2 = 1$ , whereas a purely random choice leads to  $Y_2 \sim 1/N$ .

As shown in two examples below, this problem is similar to the random energy model of spin glasses [36]. Whereas  $Y_2 \rightarrow 0$  as  $N \rightarrow \infty$  in the disorder-free homogeneous case ( $k_i = \text{constant}$ ), there is a  $O(1)$ , or even unit probability that the two walkers choose the same site in strongly heterogeneous media ( $\beta < 2$ , here).

First, we note that

$$y_2 = \frac{\sum_{i=1}^N r_i^2}{(\sum_{i=1}^N r_i)^2} = \sum_{i=1}^N \int_0^\infty dt \, t r_i^2 e^{-tr_i} e^{-t \sum_{j \neq i} r_j}, \quad (9)$$

where the identity  $A^{-2} = \int_0^\infty dt \, t \exp(-tA) / \Gamma(2)$  has been used. Since the weights are independently distributed among targets, then

$$Y_2 = \sum_{i=1}^N \int_0^\infty dt \, t \langle r_i^2 e^{-tr_i} \rangle \prod_{j \neq i} \langle e^{-tr_j} \rangle. \quad (10)$$

We analyse below two simple choices for the preference function  $f(x)$ .

### 6.1. Exponential preference

If  $f(x) = \exp(x/T)$ , where  $T$  is a constant, then  $r_i = \exp(k_i/iT)$  and the deterministic case is recovered as  $T \rightarrow 0$  (zero ‘temperature’). At finite  $T$ , we decompose (10) into

$Y_2 = \int_0^a dt(\dots) + \int_a^\infty dt(\dots) \equiv Y_2^< + Y_2^>$ , with  $a$  being a constant  $\ll 1$ . To evaluate the integrand of the first integral, we first approximate  $\exp(-tr_i)$  to 1 if  $r_i < 1/t$ , and to 0 otherwise. Therefore,

$$\begin{aligned} \langle e^{-tr_i} \rangle &\simeq \int_1^{-iT \ln t} p(k) dk = 1 - (-iT \ln t)^{-\beta+1} \\ &\simeq \exp[-(-iT \ln t)^{-\beta+1}], \end{aligned} \quad (11)$$

an expansion valid for small  $t$  only. Given that  $\langle r_j^2 e^{-tr_j} \rangle = \frac{\partial^2}{\partial t^2} \langle e^{-tr_j} \rangle$ ,

$$\langle r_j^2 e^{-tr_j} \rangle = \frac{(\beta-1)(jT)^{-\beta+1}}{t^2(-\ln t)^\beta} \exp[-(-iT \ln t)^{-\beta+1}], \quad (12)$$

at leading order as  $t \rightarrow 0$ . Substituting (11) and (12) into (10) and making the change  $t \rightarrow y = \sum_{j=1}^N (-jT \ln t)^{-\beta+1}$ , one obtains

$$Y_2^< = 1 - \exp \left[ -(-T \ln a)^{-\beta+1} \sum_{j=1}^N j^{-\beta+1} \right]. \quad (13)$$

If  $\beta < 2$  (superabundant regime in 1d [26]), the above sum diverges with  $N$ :

$$Y_2^< \simeq 1 - \exp \left[ -(-T \ln a)^{-\beta+1} \frac{N^{2-\beta}}{2-\beta} \right] \rightarrow 1 (N \rightarrow \infty). \quad (14)$$

Thus, one may conclude that

$$Y_2 = 1 \quad (1 < \beta < 2), \quad (15)$$

like in the deterministic case, for sufficiently large systems at any finite noise intensity  $T$ : the most attractive target at  $i^*$  completely dominates the sum  $\sum_i r_i$  and the probability of choosing  $i^*$  among the  $N$  targets tends to unity even at finite  $T$ .

On the other hand, in the disorder-free case (homogeneous medium or  $\beta \rightarrow \infty$ ),

$$Y_2 = \frac{\sum_{i=1}^N e^{\frac{2}{iT}}}{\left(\sum_{i=1}^N e^{\frac{1}{iT}}\right)^2} \sim \frac{1}{N} \rightarrow 0, \quad (16)$$

for any finite  $T > 0$ . Hence, all targets become practically equally attractive and each walker chooses one of them essentially at random (delocalized regime).

For finite  $\beta > 2$ , numerical results indicate that  $Y_2$  remains finite as  $N \rightarrow \infty$ , although it decays rapidly with  $\beta$ , see figure 7. The values of  $Y_2$  plotted are practically unchanged for  $N = 10^6$  and  $N = 10^7$ . Hence, the finite values obtained for  $\beta > 2$  cannot be attributed to finite  $N$  effects.

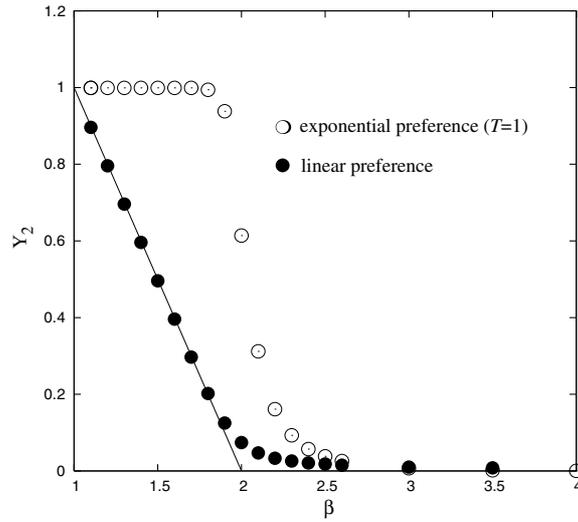
## 6.2. Linear preference

Another case of interest is the linear preferential choice  $f(x) = x$ , i.e.,  $r_i = k_i/i$ . It is exactly soluble and closely analogous to the random energy model [36]. In the disorder-free case ( $k_i = 1$ ),  $Y_2$  reads

$$Y_2 = \frac{\sum_{i=1}^N i^{-2}}{\left(\sum_{i=1}^N i^{-1}\right)^2} \sim \frac{\zeta(2)}{(\ln N)^2} \rightarrow 0, \quad (17)$$

at large  $N$ .

In the presence of disorder with  $\beta > 2$ , one has  $\langle r_i \rangle = j^{-1}(\beta-1)/(\beta-2) < \infty$ . Since only the small  $t$  behaviour contributes in the product in (10), then  $\langle \exp(-tr_i) \rangle \simeq 1 - t \langle r_i \rangle \simeq$



**Figure 7.** Numerically computed probability  $Y_2$  (averaged over  $10^4$  disordered media) that two independent walkers choose the same site in the one-dimensional problem with  $N = 10^7$ . The straight line is relation (21).

$\exp(-t\langle r_i \rangle)$  and  $\langle r_i^2 \exp(-tr_i) \rangle \simeq \langle r_i \rangle^2 \exp(-t\langle r_i \rangle)$ . Substituting these expressions in (10), one finds

$$Y_2 \sim \int_0^\infty du u \left( \sum_{i=1}^N i^{-2} \right) e^{-(\sum_{i=1}^N i^{-1})u} = \frac{\sum_{i=1}^N i^{-2}}{(\sum_{i=1}^N i^{-1})^2} \rightarrow 0, \quad (18)$$

the same result as the disorder-free case (17). Therefore, disorder is irrelevant in heterogeneous media with  $\beta > 2$ .

For  $1 < \beta < 2$ ,  $\langle r_i \rangle = \infty$ . It is convenient to write

$$\langle e^{-tr_i} \rangle = 1 - (\beta - 1) \int_1^\infty dk k^{-\beta} [1 - \exp(-tk/i)]. \quad (19)$$

Since  $\int_0^1 dk k^{-\beta} [1 - \exp(-tk/i)]$  is integrable and tends to 0 as  $t \rightarrow 0$ , the lower bound 1 can be replaced by 0 in the integral of (19). One then obtains,

$$\begin{aligned} \langle e^{-tr_i} \rangle &= 1 - (\beta - 1)(t/i)^{\beta-1} \int_0^\infty dx x^{-\beta} (1 - e^{-x}) \\ &\simeq \exp[-(\beta - 1)(t/i)^{\beta-1}(-\Gamma(1 - \beta))], \end{aligned} \quad (20)$$

for small enough  $t$ . By using (20) and the identity  $\langle r_j^2 e^{-tr_j} \rangle = \frac{\partial^2}{\partial t^2} \langle e^{-tr_j} \rangle$ , (10) can be integrated and gives the simple result:

$$Y_2 = 2 - \beta \quad (1 < \beta < 2). \quad (21)$$

The results are confirmed numerically in figure 7. Like in the previous exponential preference,  $Y_2$  is non-vanishing in the limit of large system sizes for strongly heterogeneous media ( $\beta < 2$ ). This result is asymptotically identical to that obtained for the second moment of the weights of the microscopic states in the partition function of the random energy model at low temperatures [36]. It is also identical to the  $Y_2$  parameter for a sum of positive Lévy variables [37], a problem equivalent to setting  $r_i = k_i$  instead of  $r_i = k_i/i$  in (10). This shows that

in strongly disordered environments, the distance travelled to the best target does not play a major role compared to the target size in the choice process. However, since  $Y_2 < 1$ , noise is not completely irrelevant: there is still a finite probability that the two independent walkers choose a different target.

## 7. Discussion and conclusions

We have shown that a ‘knowledgeable’ walker following simple deterministic rules in a medium composed of targets with heterogeneous weights describes trajectories characterized by multiple length scales. For a particular distribution of target weights, the sojourn lengths are maximally fluctuating and distributed in a Lévy-like manner, as  $P(l) \sim l^{-\alpha}$  with  $\alpha \simeq 2$ . As shown in [25], this model reproduces observations of spider monkeys foraging patterns, where  $\alpha \simeq 2$  has been measured. These monkeys feed on trees whose size distribution is broad, approximately an inverse power law with exponent  $\beta \simeq 2.6$  [25], not far from the special value  $\beta_c \simeq 3$  found in the model.

The type of processes modelled here (but see also [35] for a similar approach) also represent a useful alternative to merely random-walk descriptions of human travels: humans use (mental) maps, take non-random decisions and concentrate their activity very heterogeneously in space. Qualitatively, our model could describe the movement patterns of hunter–gatherers [15] or the diffusion process of bank notes mediated by the travels of modern humans at the scale of a country [16]. The former system is empirically well described by a step length distribution with exponent  $\alpha \simeq 2$  [15], and the latter by  $\alpha \simeq 1.6$  [16]. In [16], long enough steps connect different cities and the power-law behaviour could be induced by the broad distribution of city sizes, that can be approximated by Zipf’s law [30], i.e. relation (1) with  $\beta = 2$ . Even though our results agree only qualitatively with these observations (no scaling behaviour in  $P(l)$  is actually found here for  $\beta = 2$ ), the present framework, appropriately modified, may provide a promising alternative for the description of this process. Future research should consider more sophisticated decision rules and systems of targets not uniformly distributed in space.

On a theoretical point of view, the model proposed exhibits rich dynamical features in spite of its simplicity. We find, for example, that not every scale-free weight distribution induces walks with scale-free properties. Besides, the fact that the mean field approximation worked out in [26] is not very accurate, suggests that memory effects (arising the avoidance of previously visited sites) play a crucial role. The special exponent value  $\alpha = 2$  found in simulations is actually not well understood.

We also present arguments showing that in the presence of strong disorder (small values of  $\beta$  here), the addition of stochasticity in the choice rules should not affect the main properties of the walks, provided that the probability of visiting a site increases sufficiently rapidly with the site attractiveness. This situation is similar to the lack of self-averaging in spin glasses at low temperature [37]. As shown by our calculation of  $Y_2$  in the one-dimensional case, noise should not affect the distribution of sojourn length  $P(l)$  much, thus explaining *a posteriori* the results found in [25] in a variant of the model including forager mistakes. The analytical results presented here being limited to a single step process, it would be interesting to further investigate whether there is still a finite probability to find two (or  $M$ ) independent walkers at the same site after many steps. Such a probability does remain asymptotically finite in several random walk models in disordered media, the remarkable Golosov localization phenomenon [38].

A system of many non-interacting travelling agents with mental map is susceptible to make contact (when they occupy a same site) much more frequently than random walkers.

The results above suggest that such contact networks could be, in some cases, controlled by the quenched randomness of the medium in spite of the stochasticity of the agent's decisions. Such robustness could have important social implications, for instance, for the formation of social networks or the spreading of diseases [39].

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